

**Si torna indietro**

(Teoria a pagina 445)

**Scomposizioni***Scomporre in fattori i seguenti polinomi; raccogliere il fattore comune e riconoscere i prodotti notevoli.*

**598**  $a^2b - b^3$   $[b(a+b)(a-b)]$

**599**  $2x^2 - 2x^4$   $[2x^2(1+x)(1-x)]$

**600**  $2x^2 - 32x^6$   $[2x^2(1+4x^2)(1+2x)(1-2x)]$

**601**  $2x - 2x^9$   $[2x(1+x^4)(1+x^2)(1+x)(1-x)]$

**602**  $3x^5 - 48x$   $[3x(x^2+4)(x+2)(x-2)]$

**603**  $x^8 - x^6y^2$   $[x^6(x+y)(x-y)]$

**604**  $5ab^2 - 5ac^2$   $[5a(b+c)(b-c)]$

**605**  $x^3 - x^7$   $[x^3(1+x^2)(1+x)(1-x)]$

**606**  $a^8 - a^{16}$   $[a^8(1+a^4)(1+a^2)(1+a)(1-a)]$

**607**  $x^4 - 4x^3 + 4x^2$   $[x^2(x-2)^2]$

**608**  $bx^2 - 2bx + b$   $[b(x-1)^2]$

**609**  $ab^2 + 2ab + a$   $[a(b+1)^2]$

**610**  $3x^2 + 12y^2 - 12xy$

**611**  $8a^2 + 18 - 24a$

**612**  $4x^7y^5 - 24x^6y^6 + 36x^5y^7$

**613**  $4abx^2 - 32a^2bx + 64a^3b$

**614**  $4y^4 - 8y^3z + 4y^2z^2$

**615**  $18a^6b^6 + 48a^5b^4 + 32a^4b^2$

**616**  $a^3x - 3a^2x + 3ax - x$   $[x(a-1)^3]$

**617**  $16a^4 + 24a^3 + 12a^2 + 2a$   $[2a(2a+1)^3]$

**618**  $81a^4 - 27a^3b + 3a^2b^2 - \frac{ab^3}{9}$   $\left[3a\left(3a - \frac{b}{3}\right)^3\right]$

**619**  $3a^7x^2 - 27a^5x^3 + 81a^3x^4 - 81ax^5$   $[3ax^2(a^2 - 3x)^3]$

*Scomporre in fattori i polinomi eseguendo raccoglimenti totali o parziali e, dove è possibile, riconoscendo i prodotti notevoli.*

**620**  $a^2(1-3a) - b^2(1-3a)$   $[(1-3a)(a+b)(a-b)]$

**621**  $(x+y)^2 - 3(x+y)$   $[(x+y)(x+y-3)]$

**622**  $(3a-b)^2 + (3a-b)5a - (3a-b)$   $[(3a-b)(8a-b-1)]$

**623**  $a^2 + ax - x - a$   $[(a+x)(a-1)]$

**624**  $4a^2 - 4a - ax + x$   $[(a-1)(4a-x)]$

**625**  $6x^3 + 3x^2 - 4x - 2$   $[(2x+1)(3x^2-2)]$

**626**  $8a^2 - 12a + 10ab - 15b$   $[(2a-3)(4a+5b)]$

**627**  $x^2 + 3x - 2xy - 6y$   $[(x+3)(x-2y)]$

**628**  $4x^2 - xy - 4x + y$   $[(x-1)(4x-y)]$

**629**  $ab - ax + b - x$   $[(a+1)(b-x)]$

**630**  $ax - ay + bx - by - 2x + 2y$   $[(x-y)(a+b-2)]$

**631**  $ab - a - 2b^2 + 2b$   $[(a-2b)(b-1)]$

**632**  $2a^2 + 2ab + a + b$   $[(a+b)(2a+1)]$

**633**  $x^2y^2 - x^2y - xy^2 + xy$   $[xy(x-1)(y-1)]$

**634**  $4x^2y + 6xy^2 - 6x^2 - 9xy$   $[x(2y-3)(2x+3y)]$

**635**  $3x^2 - 3xy + 2x - 2y$   $[(x-y)(3x+2)]$

**636**  $ab^2 + a - b^2 - 1$   $[(a-1)(b^2+1)]$

**637**  $9ax^2 + 18x^2 - ay^2 - 2y^2$   $[(3x+y)(3x-y)(a+2)]$

**638**  $a^2x^2 - a^2y^2 - b^2x^2 + b^2y^2$   $[(a+b)(a-b)(x+y)(x-y)]$

**639**  $18x^3 - 4 - 8x + 9x^2$   $[(2x+1)(3x+2)(3x-2)]$

**640**  $ab^2 - a - b^2 + 1$   $[(a-1)(b+1)(b-1)]$

**641**  $2xy + 2x - (y+1)^2$   $[(y+1)(2x-y-1)]$

**642**  $(a+b)^2 - 3ab - 3b^2$   $[(a+b)(a-2b)]$

**643**  $x^2 - y^2 + x^2y^2 - 1$   $[(y^2+1)(x+1)(x-1)]$

**644**  $a^4 + a^2 - b^4 - b^2$   $[(a^2+b^2+1)(a+b)(a-b)]$

**645**  $x^3 - x^2 - 4x + 4$   $[(x-1)(x+2)(x-2)]$

**646**  $6a^2b + 4a^2b^2 - 6ab^3 - 9ab^2$   $[ab(3+2b)(2a-3b)]$

**647**  $2x - 3y + 4x^2 - 9y^2$   $[(2x-3y)(1+2x+3y)]$

**648**  $x^2 - 4y^2 - x + 2y$   $[(x-2y)(x+2y-1)]$

**649**  $a^2 + 2ab + b^2 + 2a + 2b$   $[(a+b)(a+b+2)]$

**650**  $xy + 3y^2 - x^3 + 9xy^2$   $[(x+3y)(y-x^2+3xy)]$

**651**  $a^2 + y^2 + 2ay - 4x^2$   $[(a+y-2x)(a+y+2x)]$

**652**  $4x^2 + 12x + 9 - 9y^2$   
 $[(2x + 3 - 3y)(2x + 3 + 3y)]$

**653**  $x^2 + y^2 - 2xy - 4$   $[(x - y - 2)(x - y + 2)]$

**654**  $9x^2 - a^2 - 4b^2 + 4ab$   
 $[(3x - a + 2b)(3x + a - 2b)]$

**655**  $a^2 - 4ab + 4b^2 - 9$   $[(a - 2b + 3)(a - 2b - 3)]$

**656**  $y^2 - 25 + 10x - x^2$   $[(y - 5 + x)(y + 5 - x)]$

**657**  $4x^2 + 4x + 1 - x^2 + 6x - 9$   $[(3x - 2)(x + 4)]$

**658**  $a^4 + a^3 - a^2 - a$   $[a(a + 1)^2(a - 1)]$

**659**  $x^{2m} + 2x^m y^2 + y^4 - ax^m - ay^2$   
 $[(x^m + y^2)(x^m + y^2 - a)]$

Scomporre i seguenti polinomi, ricordando le relazioni:

$$a^3 + b^3 = (a + b) \cdot (a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b) \cdot (a^2 + ab + b^2)$$

**660**  $b^3 + 8$   $b^3 - 8$

**661**  $8b^3 - 1$   $8b^3 + 1$

**662**  $27 - x^6$   $8 + 27x^6$

**663**  $64a^3 + b^9$   $64 - y^9$

**664**  $\frac{y^3}{125} - 8$   $8y^3 + 125x^3$

**665**  $x^6 - y^6$   $x^6 + y^6$

**666**  $x^9 + y^9$   $-x^3 + 1$

**667**  $x^{12} - y^9$   $-a^3 - b^3$

**668** a.  $a^3 - b^9$   $-b^3 + 8$

**669**  $\frac{a^3}{8} - 64$   $-\frac{1}{27} - b^6$

**670**  $1 + 27z^6$   $0,064 + x^9$

**671**  $(x + y)^3 + 1 =$   
 $= [(x + y) + 1][(x + y)^2 + (x + y) + 1] = \dots$

**672**  $x^3 - (x + 1)^3$   $[-3x^2 - 3x - 1]$

**673**  $(x - 1)^3 - 1$   $[(x - 2)(x^2 - x + 1)]$

**674**  $(a + b)^3 + 1$   
 $[(a + b + 1)(a^2 + 2ab + b^2 - a - b + 1)]$

**675**  $8 - (2 + a)^3$   $[-a(12 + 6a + a^2)]$

**676**  $(k + 2)^3 - k^3$   $[2(3k^2 + 6k + 4)]$

**677**  $(x + h)^3 - x^3$   $[h(3x^2 + 3xh + h^2)]$

**678**  $(a + b)^3 - (a - b)^3$   $[2b(3a^2 + b^2)]$

**679**  $(a + 1)^3 - (1 - a)^3$   $[2a(a^2 + 3)]$

**680**  $(2x - 1)^3 - (3 - x)^3$   $[(3x - 4)(3x^2 - 3x + 7)]$

Scomporre in fattori.

**681**  $n^2 + 8n + 7$   $n^2 - 8n + 7$

**682**  $n^2 - 8n + 16$   $n^2 + 8n - 20$

**683**  $n^2 + 7n - 8$   $n^2 + 7n + 12$

**684**  $x^2 + 12x + 20$   $x^2 + 12x + 27$

**685**  $x^2 + 12x + 36$   $x^2 + 12x + 48$

**686**  $x^2 + 12x - 28$   $y^2 - 11y + 28$

**687**  $y^2 - 11y + 10$   $y^2 - 11y - 60$

**688**  $x^2 + 14x + 40$   $x^2 + 14x + 48$

**689**  $x^2 + 14x + 49$   $x^2 + 14x + 45$

**690**  $a^2 - 18a + 45$   $a^2 - 18a + 81$

**691**  $m^4 - 11m^2 + 18$

**692**  $x^2 + 12xy - 45y^2$

**693**  $x^2 + 3xy - 70y^2$

**694**  $a^2 - 14ab - 15b^2$

**695**  $m^2 - 15mn + 54n^2$

**696**  $t^2 + ts - 30s^2$

**697**  $t^2 - ts - 30s^2$

**698**  $x^4 - x^2y^2 - 56y^2$

**699**  $x^2 + xy - 90y^2$

**700**  $(x - 1)^2 + 5(x - 1) + 6$

**701**  $x^{2n} - x^n - 12$

## ➤ ESERCIZIO GUIDA

Per scomporre in fattori un polinomio qualsiasi, occorre **combinare** tutti i metodi visti. Spesso bisogna operare in modo da ricondursi ai casi che conosciamo, raccogliendo parzialmente, oppure evidenziando prodotti notevoli in parti del polinomio, in modo da raccogliere successivamente. Ne mostriamo alcuni esempi.

$$\begin{aligned} 1. (3-a)^2 + (3-a)(5+a) - (3-a) &= \text{riconosciamo il fattore comune } (3-a) \text{ e lo raccogliamo} \\ &= (3-a)[(3-a) + (5+a) - 1] = \\ &= (3-a) \cdot 7 \end{aligned}$$

$$\begin{aligned} 2. a^2 + ax + ab + bx &= \text{non c'è un fattore comune, si può solo raccogliere parzialmente } a \text{ tra} \\ &= a \cdot (a+x) + b \cdot (a+x) = \text{i primi due termini, } b \text{ tra gli altri due} \\ &= (a+x)(a+b) \end{aligned}$$

$$\begin{aligned} 3. a^2 - b^2 + a - b &= \text{conviene scomporre } a^2 - b^2, \text{ in modo da evidenziare il fattore comune} \\ &= (a+b)(a-b) + (a-b) = \text{(a-b)} \\ &= (a-b)[(a+b) + 1] \end{aligned}$$

$$\begin{aligned} 4. a^2 + 4ab + 4b^2 - a - 2b &= \text{nei primi tre termini è sviluppato un prodotto notevole. Riconoscendo,} \\ &= (a+2b)^2 - (a+2b) = \text{si evidenzia anche il fattore comune} \\ &= (a+2b)[(a+2b) - 1] \end{aligned}$$

Scomporre i seguenti polinomi usando il metodo opportuno tra quelli conosciuti.

$$702 \quad x^3 + x^2 - 9x - 9 \qquad [(x+1)(x+3)(x-3)]$$

$$703 \quad (x-4)(3-x) + (3+x)(3-x)^2 \qquad [(3-x)(5+x-x^2)]$$

$$704 \quad 15x - 5y - 9x^2 - y^2 + 6xy \qquad [(3x-y)(5-3x+y)]$$

$$705 \quad a^3 + b^3 + 3a^2b + 3ab^2 - 8 \qquad [(a+b-2)(a^2+b^2+2ab+4+2a+2b)]$$

$$706 \quad a^3 - b^3 - ab^2 + a^2b \qquad [(a-b)(a+b)^2]$$

$$707 \quad 1 - x^6 - 2x^3z^3 - z^6 \qquad [(1+x^3+z^3)(1-x^3-z^3)]$$

$$708 \quad a^3 - 27b^3 + a - 3b \qquad [(a-3b)(a^2+9b^2+3ab+1)]$$

$$709 \quad 1 - 8x^3 - y + 2xy \qquad [(1-2x)(1+4x^2+2x-y)]$$

$$710 \quad x^3 - y^3 - x^2 + y^2 \qquad [(x-y)(x^2+y^2+xy-x-y)]$$

$$711 \quad a^2 - b^2 + a - b \qquad [(a-b)(a+b+1)]$$

$$712 \quad x^3 + 4x^2 + 4x \qquad [x(x+2)^2]$$

$$713 \quad 12a^2 + 12a + 3 \qquad [a(2a+1)^2]$$

$$714 \quad 2xb - 3x - 4b + 6 \qquad [(2b-3)(x-2)]$$

$$715 \quad 18a^3 - 8ab^2 \qquad [2a(3a-2b)(3a+2b)]$$

$$716 \quad 12a^3 - 3a \qquad [3a(2a+1)(2a-1)]$$

$$717 \quad 4x^6 - 12x^5 + 9x^4 \qquad [x^4(2x-3)^2]$$

**718**  $\frac{2}{9}a^3 - 8a$

**719**  $8x^2 + 4x - 6ax - 3a$

**720**  $8x - 4x^2 + 6a - 3ax$

**721**  $25x^4 + 10x^3 + x^2$

**722**  $36x^4 - 36x^3 + 9x^2$

**723**  $4a(a+1) - 8(a+1)^2$

**724**  $32x^2 + 16x + 2$

**725**  $-8 + 12a - 6a^2 + a^3$

**726**  $49x^3 - 9xy^2$

**727**  $12a^3 - 8a^2b$

**728**  $9y^2(a+1)^2 - y^2(a-1)^2$

**729**  $4x^6 + 6x^5 - 2x^4$

**730**  $(a-1)^5(a+1) - 4(a-1)(a+1)^3$

**731**  $a^2(a+1)^2 - 2a(a+1) + 1$

**732**  $6x(a+1) - 9x^2(a+1)^2 - 1$

**733**  $50(a+1)^4 - 8(a+1)^2$

**734**  $75(a+1)^2 - 27(a-2)^2$

**735**  $a^3 + 3a^2 - 18a$

→ **736**  $x^4 - 5x^2 + 4$

→ **737**  $x^2 - 4x + 3 - ax + 3a$

→ **738**  $(x^2 - 9)^2 - x^2 - 6x - 9$

→ **739**  $x^3 + x^2 - 4xy^2 - 4y^2$

$\left[2a\left(\frac{a}{3} + 2\right)\left(\frac{a}{3} - 2\right)\right]$

$[(2x+1)(4x-3a)]$

$[(2-x)(4x+3a)]$

$[x^2(5x+1)^2]$

$[9x^2(2x-1)^2]$

$[-4(a+1)(a+2)]$

$[2(4x+1)^2]$

$[(a-2)^3]$

$[x(7x-3y)(7x+3y)]$

$[4a^2(3a-2b)]$

$[4y^2(2a+1)(a+2)]$

$[2x^4(2x^2+3x-1)]$

$[(a-1)(a+1)(a^2-4a-1)(a^2+3)]$

$[(a^2+a-1)^2]$

$[-(3ax+3x-1)^2]$

$[2(a+1)^2(5a+7)(5a+3)]$

$[3(2a+11)(8a-1)]$

$[a(a-3)(a+6)]$

$[(x-1)(x-2)(x+1)(x+2)]$

$[(x-3)(x-1-a)]$

$[(x+3)^2(x-4)(x-2)]$

$[(x+1)(x+2y)(x-2y)]$



93  $\frac{y}{y+2}$   $[y \neq -2]$

94  $\frac{(a-6) \cdot a}{6a}$   $[a \neq 0; \frac{a-6}{6}]$

95  $\frac{x^2-4}{2x+2}$   $[x \neq -1]$

96  $\frac{x^2-4}{x-2}$   $[x \neq 2; x+2]$

97  $\frac{1-x}{1+x}$   $[x \neq -1]$

98  $\frac{1-x}{3x-3}$   $[x \neq +1; -\frac{1}{3}]$

99  $\frac{x-4x^3}{2x-1}$   $[x \neq \frac{1}{2}; -x(1+2x)]$

100  $\frac{a^4+2a^3}{a^7+a^6}$   $[a \neq 0 \wedge a \neq -1; \frac{a+2}{a^3(a+1)}]$

101  $\frac{x^2-10x+25}{2x^2-50}$   $[x \neq \pm 5; \frac{x-5}{2(x+5)}]$

102  $\frac{a^2+a-6}{a^2-4a+4}$   $[a \neq 2; \frac{a+3}{a-2}]$

103  $\frac{t^2+6t+9}{t+3}$   $[t \neq -3; t+3]$

104  $\frac{2a^4-16a^3+32a^2}{4a^3-64a}$   $[a \neq 0 \wedge a \neq \pm 4; \frac{a(a-2)}{2(a+2)}]$

105  $\frac{y^4-1}{y^3-y^2+y-1}$   $[y \neq 1; y+1]$

106  $\frac{a^3-1}{a^4-a^3+3a^2-3}$   $[a \neq 1; \frac{a^2+a+1}{a^3+3a+3}]$

107  $\frac{a^2-4a+3}{a^2-6a+9}$   $[a \neq 3; \frac{a-1}{a-3}]$

108  $\frac{y^2+x^2}{y+x}$   $[x \neq -y]$

109  $\frac{a^3-3a^2+3a-1}{a^2-a}$   $[a \neq 0 \wedge a \neq 1; \frac{(a-1)^2}{a}]$

110  $\frac{8x^3-a^3}{6x-3a}$   $[x \neq \frac{a}{2}; \frac{4x^2+2ax+a^2}{3}]$

111  $\frac{x^3-y^3}{x-y}$   $[x \neq y; x^2+xy+y^2]$

112  $\frac{x^2-(a-1)^2}{x+a-1}$   $[x \neq 1-a; x-a+1]$

113  $\frac{x-5}{x^2+6x}$   $[x \neq 0 \wedge x \neq -6]$

114  $\frac{a+3}{3a^3+18a^2+27a}$   $[a \neq 0 \wedge a \neq -3; \frac{1}{3a(a+3)}]$

115  $\frac{ab}{a+b}$   $[a \neq -b]$

116  $\frac{a-b}{ab}$   $[a \neq 0 \wedge b \neq 0]$

117  $\frac{x+1}{x^2+y^2}$   $[x \neq 0 \vee y \neq 0]$

118  $\frac{x-y}{x^2+1}$   $[Q]$

119  $\frac{a^2+3a}{(a+3)(a+4)}$   $[a \neq -3 \wedge a \neq -4; \frac{a}{a+4}]$

120  $\frac{3r^2-3r^3}{r^2-r}$   $[r \neq 0 \wedge r \neq 1, -3r]$

121  $\frac{2x-2y}{x^2-xy}$   $[x \neq 0 \wedge x \neq y; \frac{2}{x}]$

Moltiplicazione, divisione, potenza

Nelle seguenti espressioni eseguire i calcoli e semplificare il risultato, discutendo per quali valori assegnati alle lettere le espressioni perdono significato.

122  $\frac{2x^2y}{3xy^2} \cdot \frac{x^3}{4y^2z}$   $[\frac{x^4}{6y^3z}]$

123  $\frac{a}{b} \cdot \frac{3a^2b^2}{a+b} \cdot \frac{b^4}{2a^3}$   $[\frac{3b^5}{2(a+b)}]$

124  $\frac{6x^4}{27x^6} \cdot \frac{10x^5}{15x}$   $[\frac{1}{3x^6}]$

$$125 \quad \frac{7x^2}{14x^5} : \frac{6x^3}{4x}$$

$$\left[ \frac{1}{3x^5} \right]$$

$$126 \quad (x+y) \cdot \frac{x+2y}{x(x+y)}$$

$$\left[ \frac{x+2y}{x} \right]$$

$$127 \quad 2x \cdot \frac{x-1}{4x-4}$$

$$\left[ \frac{x}{2} \right]$$

$$128 \quad \left( \frac{3}{xy} \right)^3 \cdot \frac{x^2y - xy^2}{(x-y)^2}$$

$$\left[ \frac{27}{x^2y^2(x-y)} \right]$$

$$129 \quad 8x^2 \cdot \frac{x-1}{4x^2-4x}$$

$$[2x]$$

$$130 \quad \frac{3x-3y}{10xy} \cdot \frac{20x^2y^2}{x^2-y^2}$$

$$\left[ \frac{6xy}{x+y} \right]$$

$$131 \quad \frac{2x+1}{x} : \frac{4x^2+4x+1}{x^2y}$$

$$\left[ \frac{xy}{2x+1} \right]$$

$$132 \quad \frac{x^2-10x+25}{2x-10} \cdot \frac{4x^2}{x^2-5x}$$

$$[2x]$$

$$133 \quad \frac{a-5}{a+2} \cdot \frac{a^2-4}{2-a}$$

$$[5-a]$$

$$134 \quad \frac{y^2-y-6}{y+3} \cdot \frac{y^2+3y}{3y-9}$$

$$\left[ \frac{y(y+2)}{3} \right]$$

$$135 \quad \left( \frac{x}{x+2} \right)^2 \cdot \left( \frac{x+2}{xy} \right)^3$$

$$\left[ \frac{x+2}{xy^3} \right]$$

$$136 \quad \frac{a^2+a}{a^2-2a+1} \cdot \frac{a^2-a}{a^2+2a+1}$$

$$\left[ \frac{a^2}{a^2-1} \right]$$

$$137 \quad (x^3-y^3) : \frac{x-y}{3x}$$

$$[3x(x^2+xy+y^2)]$$

$$138 \quad \frac{y^2-2y+1}{y+1} : (y-1)$$

$$\left[ \frac{y-1}{y+1} \right]$$

$$139 \quad \frac{y^2-2y+1}{y+1} : \left( \frac{y-1}{y+1} \right)^2$$

$$[y+1]$$

$$140 \quad \frac{x^2-6x+5}{x-1} : \frac{x-5}{x^2-2x+1}$$

$$[(x-1)^2]$$

$$141 \quad \frac{a^2-b^2}{a^2} \cdot \frac{a+b}{a-b} : \frac{a^2-b^2}{ab}$$

$$\left[ \frac{b(a+b)}{a(a-b)} \right]$$

$$142 \quad \frac{2a-3b}{ab} : (4a^2-9b^2) \cdot a^3b^2$$

$$\left[ \frac{a^2b}{2a+3b} \right]$$

$$143 \quad \left( \frac{5x-2}{x} \right)^2 : \left( \frac{1}{x^2} \right)^3 \cdot \frac{1}{5x-2}$$

$$[x^4 \cdot (5x-2)]$$

$$144 \quad (x^3-y^3) \cdot \left( \frac{x}{x-y} \right)^2 : \frac{x+y}{x-y}$$

$$\left[ \frac{x^2(x^2+xy+y^2)}{x+y} \right]$$

$$145 \quad \left( \frac{6+2x}{x^2-5x+6} : \frac{x^2+6x+9}{x^2-2x} \right) : \frac{9-x^2}{x^2}$$

$$\left[ \frac{-2x^3}{(x^2-9)^2} \right]$$

$$146 \quad \left( \frac{x^2+2x-3}{x^2-9} : \frac{x^2+x-2}{2x^2-6x} \right) \cdot \frac{x^2-x-6}{x^2-2x}$$

$$\left[ \frac{2(x-3)}{x-2} \right]$$

$$147 \quad \left( \frac{a^3+b^3}{(a+b)^3} : \frac{2}{a+b} \right)^2 : (a^2+ab+b^2)$$

$$\left[ \frac{a^2+ab+b^2}{4(a+b)^2} \right]$$

### Somme di frazioni algebriche

Calcolare il valore delle seguenti espressioni, attribuendo alle lettere il valore indicato. Discutere per quali valori assegnati alle lettere le espressioni perdono significato e ridurle a una sola frazione.

$$148 \quad x + \frac{1}{x}$$

$$x = 10$$

$$\left[ \frac{101}{100}; x \neq 0; \frac{x^2+1}{x} \right]$$

$$149 \quad \frac{1}{a+1} + \frac{1}{a-1}$$

$$a = 2$$

$$\left[ \frac{4}{3}; a \neq \pm 1; \frac{2a}{a^2-1} \right]$$

$$150 \quad \frac{x}{x+1} - \frac{x+1}{x}$$

$$x = 5$$

$$\left[ -\frac{11}{30}; x \neq 0 \wedge x \neq -1; \frac{-1-2x}{x(x+1)} \right]$$

**151**  $\frac{a}{2-a} + \frac{a+2}{a} \quad a=3$

$\left[-\frac{4}{3}; a \neq 0 \wedge a \neq 2; \frac{4}{a(2-a)}\right]$

**152**  $\frac{1}{1+\frac{1}{x}} \quad x=-4$

$\left[\frac{4}{3}; x \neq 0 \wedge x \neq -1; \frac{x}{x+1}\right]$

**153**  $\frac{1}{x-\frac{1}{x}} \quad x=\frac{1}{2}$

$\left[-\frac{2}{3}; x \neq 0 \wedge x \neq \pm 1; \frac{x}{x^2-1}\right]$

Dopo aver trovato il comune denominatore delle frazioni, svolgere le seguenti espressioni (trascurare la discussione dei denominatori).

**154**  $\frac{1}{x} - \frac{1}{x-1} = \frac{x-1}{x(x-1)} - \frac{x}{x(x-1)} = \frac{-1}{x(x-1)}$

**155**  $\frac{x-1}{x+1} + \frac{1}{x} \quad \left[\frac{x^2+1}{x(x+1)}\right]$

**156**  $x+2 + \frac{1}{x} \quad \left[\frac{(x+1)^2}{x}\right]$

**157**  $x - \frac{2}{x-1} \quad \left[\frac{x^2-x-2}{x-1}\right]$

**158**  $\frac{x}{y} - \frac{y}{x} \quad \left[\frac{x^2-y^2}{xy}\right]$

**159**  $\frac{a+x}{ax} - \frac{b+2x}{bx} + \frac{2(a-2b)}{ab} \quad \left[-\frac{3}{a}\right]$

**160**  $\frac{a+b}{ab} + \frac{2(a-2c)}{ac} - \frac{c+2b}{bc} \quad \left[-\frac{3}{a}\right]$

**161**  $a-b + \frac{b^2}{a+b} \quad \left[\frac{a^2}{a+b}\right]$

**162**  $\frac{3y}{2x-y} - 1 \quad \left[\frac{2(2y-x)}{2x-y}\right]$

**163**  $x - \frac{x+2}{x-1} - 2 \quad \left[\frac{x(x-4)}{x-1}\right]$

**164**  $\frac{5a-2b}{b-3a} - 1 \quad \left[\frac{8a-3b}{b-3a}\right]$

**165**  $\frac{6a}{9a^2-1} - \frac{1}{3a+1} - \frac{1}{3a-1} \quad [0]$

**166**  $\frac{2x}{1-x^2} - \frac{x}{x^2-1} \quad \left[\frac{3x}{1-x^2}\right]$

**167**  $\frac{a}{a^2-1} - \frac{3a}{1-a} \quad \left[\frac{3a^2+4a}{a^2-1}\right]$

**168**  $\frac{4}{xy} + \frac{1}{y} - \frac{x}{xy-2y^2} + \frac{2}{x-2y} \quad \left[\frac{4}{xy}\right]$

**169**  $\frac{x-y}{x^2+2xy+y^2} - \frac{1}{3x+3y} \quad \left[\frac{2x-4y}{3(x+y)^2}\right]$

**170**  $\frac{1}{x-1} - \frac{1+x}{x^2-2x+1} \quad \left[\frac{-2}{(x-1)^2}\right]$

**171**  $\frac{2}{a^2-9a+14} + \frac{1}{7a-a^2} \quad \left[\frac{a+2}{a(a-2)(a-7)}\right]$

**172**  $\frac{5a-9}{a^2-4a+3} - \frac{2}{a-1} - \frac{4}{a-3} \quad \left[-\frac{1}{a-3}\right]$

**173**  $\frac{2a-a^2}{ax^2+ax+a} - \frac{2-ax}{x^3-1} \quad \left[\frac{2x-4+a}{x^3-1}\right]$

**174**  $\frac{2x-y}{xy} - \frac{x+2y}{x^2+xy} + \frac{x}{xy+y^2} \quad \left[\frac{3(x-y)}{xy}\right]$

**175**  $\frac{a}{ab+b^2} - \frac{a+2b}{a^2+ab} + \frac{2a-b}{ab} \quad \left[\frac{3(a-b)}{ab}\right]$

**176**  $\frac{1}{x+h} - \frac{1}{x} \quad \left[-\frac{1}{x(x+h)}\right]$

**177**  $\frac{1}{x^2-x} - \frac{1}{x} \quad \left[\frac{1}{x}\right]$

$$178 \quad \frac{\frac{1}{a} + \frac{1}{b}}{a + b}$$

$$\left[ \frac{1}{ab} \right]$$

$$180 \quad \frac{a + \frac{1}{a}}{a^2 - \frac{1}{a^2}}$$

$$\left[ \frac{a}{a^2 - 1} \right]$$

$$179 \quad \frac{\frac{a}{b} - \frac{b}{a}}{\frac{1}{a} - \frac{1}{b}}$$

$$[-(a + b)]$$

$$181 \quad \frac{1}{1 + \frac{1}{1 + \frac{1}{a}}}$$

$$\left[ \frac{a + 1}{2a + 1} \right]$$

### Espressioni con le frazioni algebriche

Ricordando le convenzioni di precedenza tra le operazioni, ridurre le seguenti espressioni a una sola frazione algebrica, non ulteriormente semplificabile (trascurare la discussione dei denominatori).

$$182 \quad \left( \frac{2}{x} - \frac{x}{x+2} \right) \cdot \frac{(x+2)^2}{x} = \frac{2(x+2) - x^2}{x(x+2)} \cdot \frac{(x+2)^2}{x} = \frac{(2x+4-x^2)(x+2)}{x^2}$$

$$183 \quad \left( \frac{b+a}{ab} \right)^2 \cdot \frac{a-b}{a+b} - \frac{a+b}{ab} = \frac{(a+b)(a-b)}{a^2b^2} - \frac{a+b}{ab} = \frac{(a+b)(a-b-ab)}{a^2b^2}$$

$$184 \quad \left( 1 + \frac{3}{x} \right) \cdot \left( \frac{3}{x} - 1 \right)$$

$$\left[ \frac{9-x^2}{x^2} \right]$$

$$185 \quad \left( x - \frac{1}{x} \right) \cdot \frac{x+1}{x-1}$$

$$\left[ \frac{(x+1)^2}{x} \right]$$

$$186 \quad \left( \frac{a}{b} - \frac{b}{a} \right) : \frac{a+b}{ab}$$

$$[a - b]$$

$$187 \quad \left( \frac{1}{a+1} - \frac{1}{a-1} \right) \cdot \frac{a+1}{a-1}$$

$$\left[ -\frac{2}{(a-1)^2} \right]$$

$$188 \quad \left( 1 - \frac{1}{a} + \frac{1}{a^2} \right) \cdot \frac{2a^3}{1-a+a^2}$$

$$[2a]$$

$$189 \quad \frac{2-b}{2+b} \cdot \left( 1 - \frac{b}{1+b} \right)$$

$$\left[ \frac{2-b}{(2+b)(1+b)} \right]$$

$$190 \quad \frac{5(a+2)(a-2) - (2a+1)^2}{(a+3)(a-4)}$$

$$\left[ \frac{a-7}{a-4} \right]$$

$$191 \quad \frac{b+5}{b^2-16} \cdot \left( 1 - \frac{9}{b+5} \right)$$

$$\left[ \frac{1}{b+4} \right]$$

$$192 \quad \left( \frac{9}{x+7} - 1 \right) \cdot \frac{x+7}{4-x^2}$$

$$\left[ \frac{1}{2+x} \right]$$

$$193 \quad \left( \frac{x}{x-1} + \frac{x}{1+x} \right) : \left( 1 - \frac{1}{1-x^2} \right)$$

$$[2]$$

$$194 \quad \left( 1 + \frac{3}{x} \right) \cdot \left( \frac{3}{x} - 1 \right) \cdot \left( \frac{x+1}{x^2-3x} - \frac{x-1}{x^2+3x} \right)$$

$$\left[ -\frac{8}{x^2} \right]$$



$$195 \quad \left( \frac{1}{x-1} + \frac{2x}{1-x^2} \right) : \frac{x}{x+1} \quad \left[ -\frac{1}{x} \right]$$

$$196 \quad \left( \frac{2a}{1-a^2} + \frac{1}{a-1} \right) : \frac{a}{a+1} \quad \left[ -\frac{1}{a} \right]$$

$$197 \quad \left( \frac{1}{x^3} + \frac{1}{y^3} \right) \cdot \left( \frac{1}{x+y} - \frac{x+y}{x^2-xy+y^2} \right) \quad \left[ -\frac{3}{x^2y^2} \right]$$

$$198 \quad \left( \frac{3a-1}{2a-1} + \frac{a+1}{1+2a} \right) \cdot \frac{1-4a^2}{2a^2} \quad \left[ \frac{4a^2+a-1}{a^2} \right]$$

$$199 \quad \left( \frac{a-2}{ax-a+x-1} - \frac{a+2}{ax-a-x+1} \right) \cdot \left( \frac{1}{a} - a \right) \quad \left[ \frac{6}{x-1} \right]$$

$$200 \quad \left( \frac{1}{a} - \frac{1}{b} \right) \cdot \left( \frac{b}{a-b} + \frac{a}{a+b} \right) \cdot \left( \frac{1}{b} - \frac{b-a}{a^2+b^2} \right) \quad \left[ -\frac{1}{b^2} \right]$$

$$201 \quad \frac{3}{x^2+3x+2} : \left( \frac{x-2}{x^2+4x+3} - \frac{x-1}{x^2+5x+6} \right) \quad [-x-3]$$

$$202 \quad \left( a + \frac{a^2-3ab}{a+b} \right) : \left( \frac{a}{a+b} + \frac{a}{a-b} - \frac{2ab}{a^2-b^2} \right) \quad [a-b]$$

$$203 \quad \left( \frac{3}{9-6x+x^2} + \frac{1}{3-x} \right)^2 : \frac{36-12x+x^2}{9-x^2} \quad \left[ \frac{3+x}{(3-x)^3} \right]$$

$$204 \quad \left( \frac{a}{b} + \frac{b}{a} + 1 \right) \cdot \left( \frac{1}{a-b} - \frac{a-b}{a^2+ab+b^2} - \frac{b^2}{a^3-b^3} \right) \quad \left[ \frac{3a-b}{a(a-b)} \right]$$

$$205 \quad \left( \frac{x}{x+3} + \frac{1}{x+1} - \frac{x^2+1}{x^2+4x+3} \right) : \frac{6x}{x^2+6x+9} \quad \left[ \frac{x+3}{3x} \right]$$

$$206 \quad \left( \frac{c^2-3cd}{c+d} + c \right) : \left( \frac{c}{c-d} - \frac{2cd}{c^2-d^2} + \frac{c}{c+d} \right) \quad [-d]$$

$$207 \quad \left( \frac{b-x}{ab-bx+ax-x^2} + \frac{b-x}{ab+bx+ax+x^2} \right) \cdot \left( \frac{2x}{b-x} + 1 \right) \cdot \left( 1 - \frac{x^2}{a^2} \right) \quad \left[ \frac{2}{a} \right]$$

$$208 \quad \left( \frac{b-1}{b^2+4b+3} - \frac{b+3}{b^2-2b-3} \right) : \left( \frac{2}{b+3} + \frac{3}{b-3} \right) \quad \left[ -\frac{2}{b+1} \right]$$

$$209 \quad \left( \frac{x^3-25x}{x^2+8x+15} \right) : \left( \frac{x}{2(x-3)} - \frac{2}{x-3} + \frac{6+x}{(x+3)(x-3)} \right) \quad \left[ \frac{2(x-3)(x-5)}{x+1} \right]$$

$$210 \quad \left( \frac{2a+b}{a} - \frac{a^2+b^2}{ab} \right) : \left( \frac{4b}{a} - \frac{a}{b} \right) + \frac{2b}{a+2b} \quad [1]$$

$$211 \quad \frac{x^2}{2-x} \cdot \left( \frac{1}{x} - \frac{2}{x^2} \right) \cdot \left( \frac{2x^2+1}{x^2-x} - \frac{x}{x-1} + \frac{2}{1-x} \right) \quad \left[ \frac{1-x}{x} \right]$$



## TEST DI AUTOVALUTAZIONE 1

TEMPO PER LA PROVA: 30 MINUTI

1. Indica quale tra le seguenti espressioni simboliche traduce la frase:  
*Il quadrato della somma dei reciproci di due numeri positivi a e b.*

[a]  $\frac{1}{(a+b)^2}$     [b]  $\frac{1}{a^2} + \frac{1}{b^2}$     [c]  $\left(\frac{1}{a} + \frac{1}{b}\right)^2$     [d]  $\frac{1}{a^2 + b^2}$

2. Quale tra le seguenti espressioni perde significato per  $x = -3$ ?

[a]  $\frac{x+3}{x}$     [b]  $\frac{3}{5+15x}$     [c]  $\frac{x}{3x+1}$     [d]  $x^2-9$     [e]  $\frac{x}{3+x}$

3. La frazione  $\frac{2x+1}{x}$  per  $x=0$ :

[a] vale 0    [b] non ha significato    [c] vale 1    [d] è indeterminata

4. Semplifica la frazione  $\frac{x^3 - 2x^2 - 3x}{x^2(x^2 - 6x + 9)}$

5. Trasforma le seguenti frazioni in frazioni equivalenti:

a.  $\frac{4}{x^2} = \frac{\dots}{x^3 - x^2}$     b.  $\frac{x}{x-4} = \frac{x^2 + 5x}{\dots}$

Calcola e semplifica le seguenti espressioni:

6.  $\frac{2x-5y}{3y-1} - 1$

8.  $\frac{2(x-2y)}{xy} - \frac{y+2a}{ay} + \frac{x+a}{ax}$

7.  $a - 2 - \frac{a+2}{a-1}$

9.  $\left(1 - \frac{11}{x+6}\right) \cdot \frac{x+6}{x^2-25}$

## Equazioni fratte

Dopo aver individuato l'insieme di definizione, risolvere le seguenti equazioni fratte.

249  $\frac{3}{2x+1} = 5$      $\left[x \neq -\frac{1}{2}; x \neq -\frac{1}{5}\right]$

250  $\frac{2}{3x+1} = \frac{1}{x}$      $\left[x \neq 0 \wedge x \neq -\frac{1}{3}; x \neq -1\right]$

251  $\frac{8}{5x-4} = \frac{5}{3x-1}$      $\left[x \neq \frac{4}{5} \wedge x \neq \frac{1}{3}; x \neq 12\right]$

252  $\frac{y+1}{y-2} = \frac{y-1}{y-3}$      $[y \neq 2 \wedge y \neq 3; y = 5]$

253  $\frac{2z+5}{4z+1} = \frac{3z+5}{6z-1}$      $\left[z \neq -\frac{1}{4} \wedge z \neq \frac{1}{6}; z = 2\right]$

254  $\frac{1}{10} - \frac{x-3}{5x-5} = \frac{1+4x}{2x-2}$      $[x \neq 1; x \neq 0]$

255  $\frac{3x}{x+1} - \frac{x+1}{x} = 2$      $\left[x \neq 0 \wedge x \neq -1; x \neq -\frac{1}{4}\right]$

256  $\frac{1}{x} - 1 = \frac{x}{3-x}$      $\left[x \neq 0 \wedge x \neq 3; x = \frac{3}{4}\right]$

257  $\frac{1+2x}{x} - \frac{6x}{3x-1} + \frac{4}{x-3x^2} = 0$      $\left[x \neq 0 \wedge x \neq \frac{1}{3}; x = 5\right]$

258  $\frac{1}{x-1} - \frac{3}{x+2} = \frac{1}{x^2+x-2}$      $[x \neq 1 \wedge x \neq -2; x = 2]$

$$259 \quad \frac{1}{2x-1} - \frac{1}{2x+1} = \frac{3}{4x^2 - 4x + 1}$$

$$\left[ x \neq \pm \frac{1}{2}; x = -\frac{5}{2} \right]$$

$$260 \quad \frac{4}{2x-3} + \frac{6}{3x-2x^2} = \frac{2}{x}$$

$$\left[ x \neq 0 \wedge x \neq \frac{3}{2}; \text{ind.} \right]$$

$$261 \quad \frac{x}{x-3} - \frac{1-x}{3x-x^2} = 1 + \frac{1}{x}$$

$$[x \neq 0 \wedge x \neq 3; x = -4]$$

$$262 \quad \frac{x+3}{x-2} - \frac{x+4}{x+2} = \frac{2(x+7)}{x^2-4}$$

$$[x \neq \pm 2; x = 0]$$

$$263 \quad \frac{3}{2x-3} + \frac{1}{2x^2-3x} = \frac{3}{2x}$$

$$\left[ x \neq 0 \wedge x \neq \frac{3}{2}; \text{imp.} \right]$$

$$264 \quad \frac{1}{3-2x} + \frac{3}{2x^2-3x} + \frac{5}{x} = 0$$

$$\left[ x \neq 0 \wedge x \neq \frac{3}{2}; x = \frac{4}{3} \right]$$

$$265 \quad \frac{1}{x-2} + \frac{1}{x+2} = \frac{1}{x-1} + \frac{x-5}{x^2+x-2}$$

$$\left[ x \neq \pm 2 \wedge x \neq 1; x = \frac{6}{5} \right]$$

$$266 \quad \frac{1-x}{x+3} + \frac{2x}{x-2} = \frac{x^2+5(x-2)}{x^2+x-6}$$

$$[x \neq -3 \wedge x \neq 2; x = -2]$$

$$267 \quad \frac{1+x}{x+2} + \frac{x+1}{x-2} = \frac{2(x^2+2)}{x^2-4} - \frac{1}{x+2}$$

$$[x \neq \pm 2; x = 2, \text{ non acc.}]$$

$$268 \quad \frac{x-5}{x-1} - \frac{x-1}{x-5} = \frac{x-30}{x^2-6x+5}$$

$$[x \neq 1 \wedge x \neq 5; x = 6]$$

$$269 \quad \frac{\frac{1}{5}+x}{x+\frac{2}{5}} + \frac{x-\frac{3}{5}x}{\frac{4}{25}-x^2} + \frac{\frac{1}{5}-x}{\frac{2}{5}-x} = 2$$

$$\left[ x \neq \pm \frac{2}{5}; \text{imp.} \right]$$

$$270 \quad \frac{2}{x^2+1-2x} + \frac{3}{x^2-1} + \frac{x^2-5}{(x+1)(x^2-2x+1)} = \frac{1}{x+1}$$

$$[x \neq \pm 1; x = 1, \text{ non acc.}]$$

$$271 \quad \frac{3}{1-x^2} - \frac{2}{x^2-2x+1} = \frac{1}{x+1} - \frac{5+x^2}{x^3-x^2-x+1}$$

$$\left[ x \neq \pm 1; x = \frac{5}{3} \right]$$

$$272 \quad \frac{x}{x^2-9} - \frac{2}{x^2} - \frac{1}{x^3+3x^2} = \frac{x+1}{x^2+3x} - \frac{5}{3x^2-x^3}$$

$$[x \neq \pm 3 \wedge x \neq 0; x = 2]$$

$$273 \quad \frac{2}{x+1} + \frac{3x}{x^2-1} + \frac{6}{1-x} = \frac{1}{x+1}$$

COMPLETO?

$$\left[ x \neq \pm 1; x = -\frac{7}{2} \right]$$

$$274 \quad \frac{x-1}{x-\frac{3}{5}} - 1 = \frac{1-\frac{3}{5}}{x+\frac{3}{5}}$$

$$\left[ x \neq -\frac{3}{5} \wedge x \neq \frac{3}{5}; x = 0 \right]$$